

**METHOD AND APPARATUS FOR RESTRUCTURING A BINARY
DECISION DIAGRAM**

BACKGROUND OF INVENTION

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NOV 14 2002

Technology Center 2100

FIELD OF INVENTION

The present invention relates to a method and apparatus for reducing the complexity of a representation of a hardware system.

DESCRIPTION OF THE PRIOR ART

The first stage in synthesizing and proving the properties of a system is a compilation process in which the system is modeled by representation as a set of functions comprising a first subset of functions which determines the value of system outputs as a function of system inputs, system states represented by state bits, and internal signals; a second subset of functions which determines the values of state bits on the next clock cycle as a function of system inputs, system states represented by state bits, and internal signals; and a third subset of functions which determines the values of internal signals as a function of system inputs, system states, and internal signals.

To enable or accelerate formal proof of the system and its properties, internal signals may be eliminated from the system model by substituting them into the functions which refer to them. In the course of this substitution, the representation of the model may become extremely large. If this occurs, it is possible to detect an explosion in the size of the representation and to suspend the substitution process while restructuring the representation to seek a reduction in size.

Typically in a compilation process, static relationships between signals in the system model can be destroyed by dynamic restructuring operations. This can lead to a further explosion later during the substitution process.

It would be advantageous to take static relationships into account during the dynamic restructuring process.

One technique of representing functions and internal signals is by the use of binary decision diagrams (BDD's). A binary decision diagram is a representation of a digital function which contains the information necessary to implement the function. The diagram is a tree-like structure having a root and plural nodes, where the root represents the digital function and the nodes are labeled with variables. Each node has two branches, one representing the assertion that the variable labeling the node is 1, and the other representing the assertion that the variable labeling the node is 0. In a BDD, "ordering" relates to the order in which variable names are encountered during traversal of the graph. Better orderings result in fewer nodes in the graph.

SUMMARY OF INVENTION

According to a first aspect of the present invention, there is provided a method for selecting an order in which to sift variables in a binary decision diagram comprising arranging the variables of the binary decision diagram on the nodes of a graph in which the nodes are labeled with the variables of the system such that the set of functions labeling leaves reachable from a node correspond to the set of functions which depend on the variables labeling the node; and traversing the graph in a depth first manner, thereby to produce a list of the labels in the selected order.

According to a second aspect of the present invention there is provided an apparatus for selecting an order in which to sift variables in a binary decision diagram comprising a first store storing bits representing the variables of the binary decision diagram; a second store; and a processor adapted to arrange the variables of the binary decision diagram in a representation of the nodes of a graph in which the nodes are labeled with the variables such that the set of functions labeling leaves reachable from a node corresponds to the set of functions which depend on the variables labeling the node; and to traverse the graph in a depth-first manner such that the processor outputs to the second store a list of labels in the selected order.

According to a third aspect of the present invention there is provided a method for restructuring a binary decision diagram representative of a hardware system, comprising arranging variables of the binary decision diagram on nodes of a graph in which the nodes are labeled with the variables of the system such that the set of

functions labeling leaves reachable from a node corresponds to the set of functions which depend on the variables labeling the node; and traversing the graph in a depth-first manner to produce a list of labels in a selected order; using the selected order, controlling sifting of each variable.

- 5 Preferably the variables are sifted one-by-one to a deepest best location. Advantageously the variables are sifted one-by-one in the selected order to a deepest best location followed by sifting in reverse order to a shallowest best location.

 According to a fourth aspect of the present invention there is provided apparatus for restructuring a binary decision diagram comprising storage circuitry for
10 storing bits representative of a set of functions as binary decision diagrams having a plurality of nodes labeled by variables; a processor for detecting a number of nodes of the binary decision diagram, and in response to such detection, arranging the variables of the binary decision diagram on the nodes of a graph in which the nodes are labeled such that the set of functions labeling leaves reachable from a node corresponds to the
15 set of functions which depend on the variables labeling the node, traversing the graph in a depth-first fashion to produce a list of labels in a selected order and using the selected order, controlling sifting of the variables of the binary decision diagrams; wherein the sifted binary decision diagram is written by the processor to the storage circuitry.

- 20 According to a fifth aspect of the present invention there is provided a method for proving properties of a hardware system comprising representing the system as binary decision diagrams having a plurality of nodes labeled by variables; substituting functions which determine variables of internal signals; arranging the variables of a binary decision diagram on the nodes of a graph in which the nodes are labeled with
25 the variables of the system such that the set of functions labeling leaves reachable from a node corresponds to the set of functions which depend on the variables labeling the node; traversing the graph in a depth-first manner to produce a list of the labels in a selected order; and using the selected order, controlling sifting of each variable.

- 30 According to a sixth aspect of the present invention there is provided apparatus for proving properties of a hardware system comprising storage circuitry for

storing bits representative of a set of functions which represent the hardware system as binary decision diagrams having a plurality of nodes labeled by variables; processor means for substituting functions which determine values of internal signals into the set of functions representing the system and detecting an increase in the
5 number of nodes of the binary decision diagram, and, in response to such detection arranging the variable of the binary decision diagram on the nodes of a graph in which the nodes are labeled with the variables of the system such that the set of functions labeling leaves reachable from a node corresponding to the set of functions which depend on the variables labeling the node, traversing the graph in a depth-first fashion
10 to produce a list of labels in the selected order, and using the selected order controlling sifting of the variables of the binary decision diagram; and further comprising a second store, wherein the sifted binary decision diagram is written by the processor means to the second store.

Preferably the number is a threshold derived from an original number of
15 nodes. Alternatively the number is the number of nodes which branches on a predetermined variable. Alternatively the number is an absolute number.

BRIEF DESCRIPTION OF THE DRAWINGS

An embodiment of the present invention will now be described with respect to
20 the following drawings in which:

Figure 1 shows a binary decision diagram for the function

$$f = x \text{ OR } y;$$

Figure 2 shows a logical diagram of a multiplexer;

Figure 3 shows a binary decision diagram for the equation

$$b_i = \text{NOT } (a_i \text{ AND } s_i);$$

Figure 4 shows the binary decision diagram for the equation

$$d = \text{NOT } (b_1 \text{ AND } b_2 \text{ AND } b_3 \dots b_n);$$

Figure 5 is an optimally ordered substitution of the equations of Figures 3 and
4;

Figure 6 shows a graph of relationships between the variables of the
30 multiplexer of Figure 2;

Figure 7 shows an example of apparatus arranged to implement an embodiment of the present invention; and

Figure 8 shows a flow diagram indicating the features of an embodiment of the present invention.

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DESCRIPTION OF THE PREFERRED EMBODIMENTS

A Binary Decision Diagram (hereinafter referred to as a BDD) is a directed acyclic graph representative of a Boolean function as a decision procedure based on the variables on which it depends. For instance, for the function:

10 $f = x \text{ OR } y,$

f can be implemented by the decision procedure “if x then true else if y then true else false”. Each of the “if...then...else...” constructs of this decision procedure can be represented as a node in a graph.

Referring to Figure 1, the first node 1 is labeled with the variable x and there are two branches from this first node, one 11 is “true” and the other 12 is “if y then true else false”. This other branch 12 leads to a second node 2 which is labeled with the variable y , which in turn has two branches 21, 22 of which one is “true” and the other is “false”.

It will be understood that although the nodes 1 and 2 are described above as being labeled with variables, nevertheless these labels could in fact refer to functions which upon evaluation would give rise to the logical values “true” or “false”.

Referring now to Figure 2, a multiplexer consists of a first set of n NAND gates 10₁-10 _{n} , each gate having two respective inputs a_1 - a_n , s_1 - s_n . The outputs, lines b_1 and b_n of the gates, are connected to an n -input NAND gate 20 having an output d .

25 Thus, in terms of a system as described in the preamble to this patent application, the multiplexer of Figure 2 has system inputs (a_1 - a_n , s_1 - s_n), internal signals (b_1 - b_n) and a system output (d). The output d is related to the internal signals b_1 - b_n by the equation:

$$d = \text{NOT } (b_1 \text{ AND } b_2 \text{ AND } b_3 \dots b_n),$$

30 and each internal signal b_i to the respective inputs a_i and s_i by the equation:

$$b_i = \text{NOT } (a_i \text{ AND } s_i).$$

Thus,

$$d = (a_1 \text{ AND } s_1) \text{ OR } (a_2 \text{ AND } s_2) \text{ OR } \dots (a_n \text{ AND } s_n).$$

Referring to Figure 3 the relationship $b_i = \text{NOT } (a_i \text{ AND } s_i)$ is shown as a binary decision diagram.

5 Figure 4 shows the binary decision diagram representation of the expression for d in terms of the internal signals b .

By inspection, there are $3n$ variables (a_i , s_i and b_i) and there are thus $(3n)!$ apparently equally good orderings possible. However, by inspection of the overall equation for the device it would be seen that a_1 and s_1 are associated together, a_2 and s_2 are associated together and so on which means that there are in fact only $n!$ orderings which are optimal for the entire system.

10 An advantage of the present invention is that it enables more information about the system as a whole to be taken into account when performing operations which would otherwise not take this information into account. Failing to take the information into account can result in following paths which do not lead to a solution, or which are highly inefficient in reaching the solution.

Figure 5 shows a binary decision diagram for the multiplexer of Figure 2 in which the respective pairs of inputs are associated together.

20 The size of a binary decision diagram is sensitive to the order in which the variables are inspected, and efficient BDD reordering is very important. One algorithm for reordering is "sifting", wherein each variable is taken in turn and the best position of it is found by trying it in every possible position of the BDD. It is then necessary to decide which variable to take first. A known and frequently successful tool for doing this is to rank the variables according to which variable labels the greatest number of nodes and then to sift in the order of ranking.

25 In the present BDD, it is clear that each variable labels a single node and thus it would not be possible using known techniques to identify a highest ranking variable. Conventionally, in such a situation, an arbitrary order for sifting would be used.

30 The present invention makes use of a function graph which is traversed to determine an order for sifting.

As used herein, a function graph is a directed acyclic graph where the leaves are labeled with functions and the nodes are labeled with sets of variables (non-empty). The only restriction put on this graph is that a variable which is in the set labeling a node is in the "cone" of all the functions at the leaves below it and no others. This restriction plus the fact that the sets of variables must be non-empty, is
5 enough to ensure that the graph is unique. The "cone" of a function is herein defined to be all those variables on which a function depends, either directly or through the intermediate signals on which it depends.

Using a function graph to define an ordering of the variables in a BDD to
10 minimize its size may be related to the register allocation technique used in software compilation in that the ordering of the variables is derived from a traversal of the function graph in such a way that no node is visited before all of its predecessors has been visited, but each node is visited as soon as all its predecessors have been visited, unless there is a race between more than one node, in which case one of the
15 competing nodes is chosen and its subgraphs traversed first.

Figure 6 shows a function graph for the multiplexer of Figure 2 having a root labeled by b_1, b_n intermediate nodes labeled by $a_1, s_1, a_2, s_2 \dots a_n, s_n$ and leaves as shown. Traversing this function graph from the top down gives the order:

$b_1, b_2 \dots b_n, a_1, s_1, a_2, s_2 \dots a_n, s_n$.

20 By using this order which is derived from static information of the system, the binary decision diagrams of (in this case) Figures 1, 3 and 4 are sifted to provide an optimal order. This order is that represented by Figure 5.

It should be noted that substitution may be effected without restructuring the BDD, while monitoring the size of the BDD. If an explosion in BDD is detected,
25 sifting is then effected on the basis of the order provided by the present invention.

Figure 7 shows an example of apparatus to implement an embodiment of the present invention. A first storage circuitry 30 stores data representative of the variables of a binary decision diagram.

The first storage circuitry has an input 32 for receiving the variables. The
30 output 34 of the first storage circuitry is coupled to an input 42 of a processor 40. The processor receives the variables via the output 34 of the first storage circuitry and

consults a function graph by arranging the variables in a representation of the nodes of the graph such that the nodes are labeled with the variables so that the set of functions labeling leaves reachable from a node corresponds to the set of functions which depend on the variables labeling the node, as shown in Figure 6. The processor then
5 traverses the graph in a depth-first manner, as indicated in Figure 6 by the arrow, to construct a list of the labels in a selected order. The list is output from the processor via output 44 which is coupled the input 52 of second storage circuitry.

Figure 8 illustrates an exemplary process for restructuring a BDD that represents a hardware system as described above. In act 1000, the hardware system
10 is represented as a set of functions that determine values of system variables (input signals, internal signals, output signals). In act 1002, the set of functions is represented as binary decision diagrams having a plurality of nodes labeled by the variables of the system. In act 1004, functions that determine the value of internal signals in the system are substituted into the set of functions representing the system.

15 When an increase in a number of nodes in the binary decision diagram is detected in act 1006, wherein the number is a threshold derived from an original number of nodes, the process proceeds to act 1008. In act 1008, in response to the detection at act 1006, processor 40 arranges the variables of the binary decision diagram on the nodes of a graph whereby the nodes are labeled by the variables. In act 1010, the
20 graph is traversed from the top down to produce a list of the labels in a selected order, and in act 1012, the variables are sifted in the selected order to reduce the size of the binary decision diagram. In act 1012, the variables are sifted one-by-one to a deepest best location and then sifted in reverse order to a shallowest best location.

While the invention has been previously shown and described with reference
25 to a preferred embodiment, it will be understood by those skilled in the art that various changes in form and detail may be made therein without departing from the spirit and scope of the invention.

What is claimed is: